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Solving Time-Consuming Global Optimization Problems with Globalizer Software System

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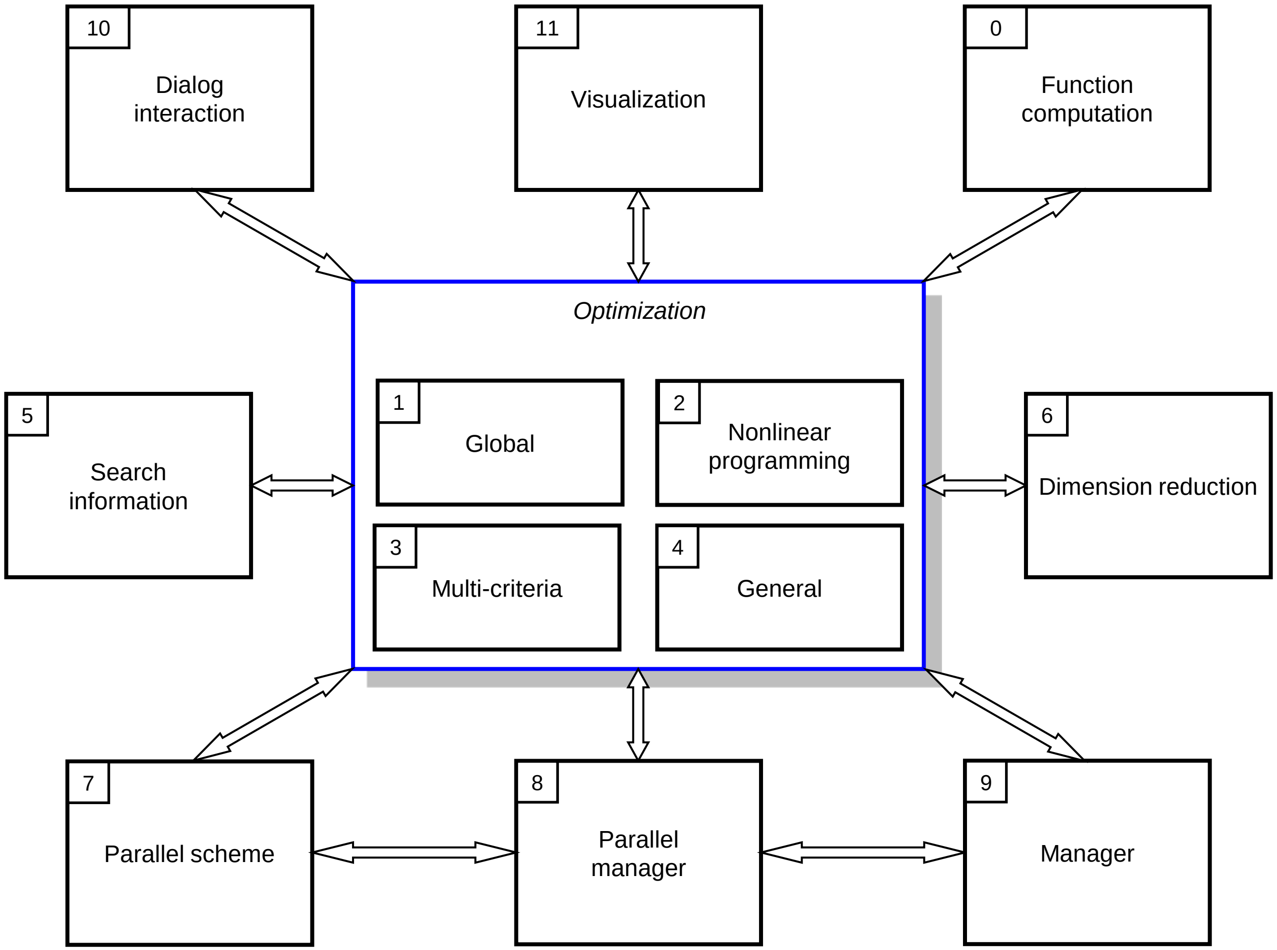
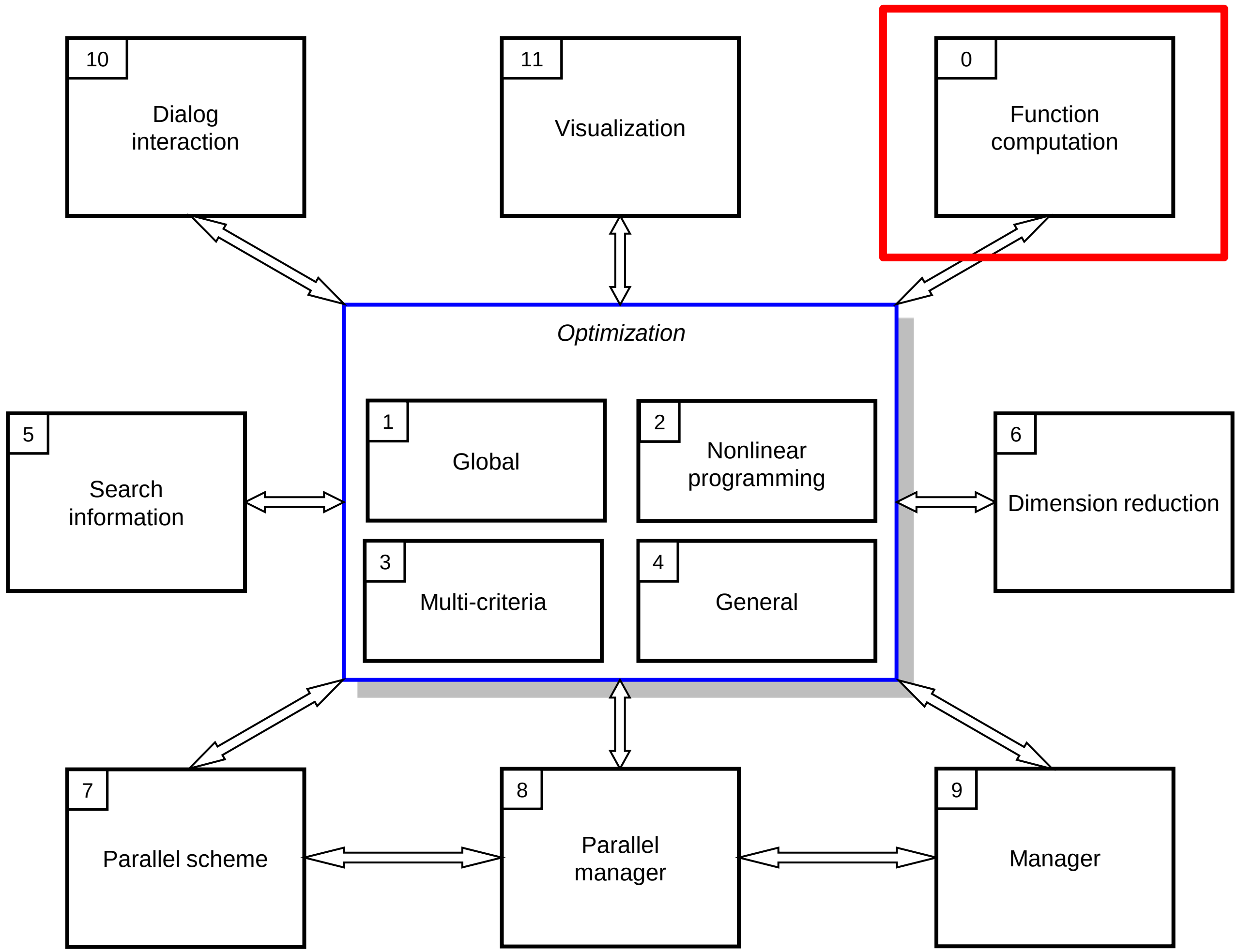
Globalizer Architecture



Globalizer Architecture



Problem statement

Consider the multidimensional optimization problem

$$\varphi(y) \to \min, \quad y = (y_1, \ldots, y_n) \qquad (1)$$

the search domain is the hyperinterval

$$D = \left\{ y \in R^N : a_i \le y_i \le b_i, \quad 1 \le i \le N \right\} \qquad (2)$$

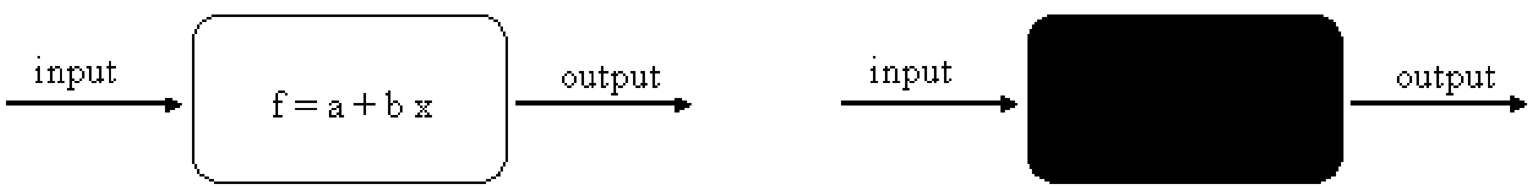
Problem statement

A priori information about the problem

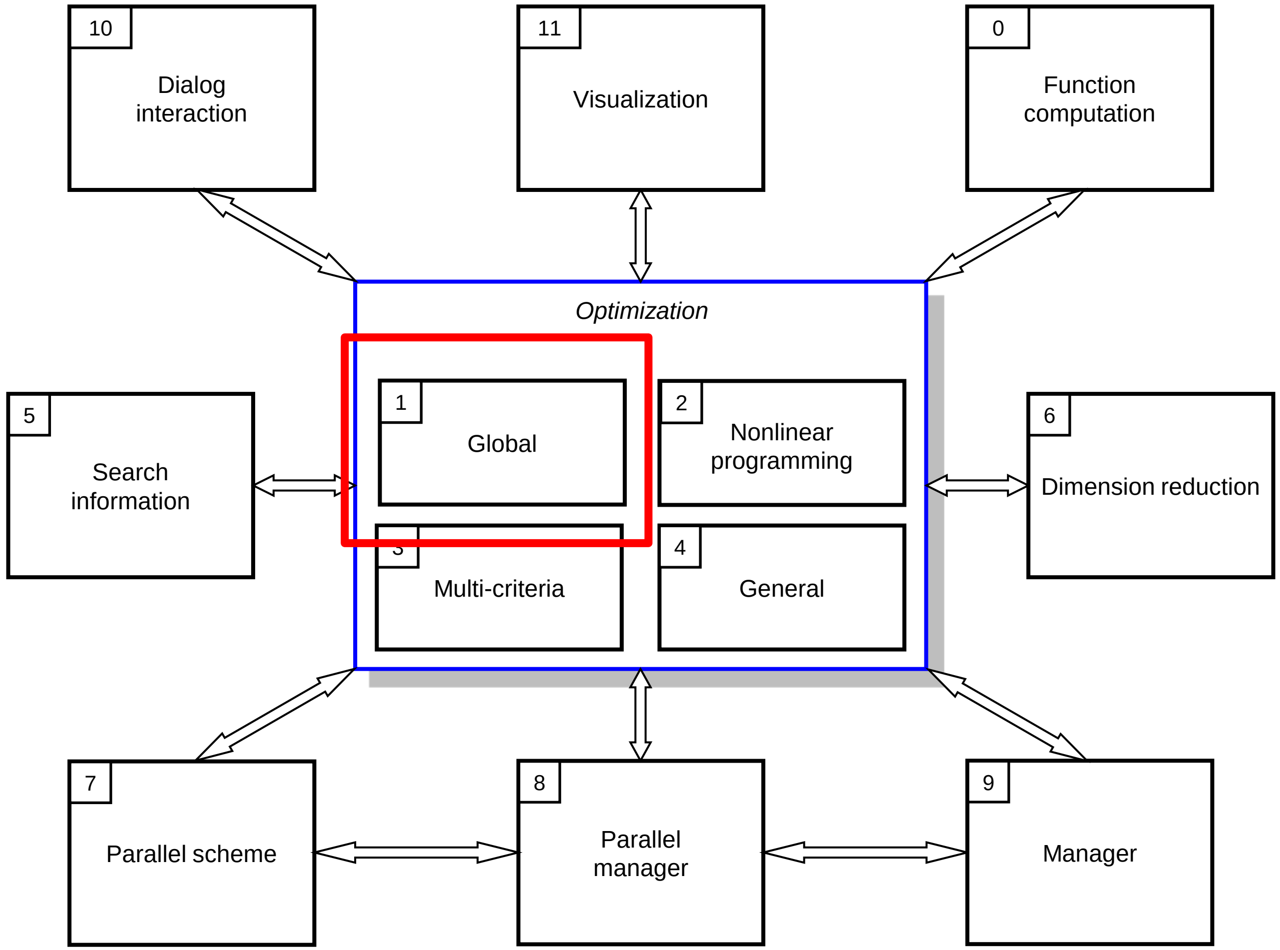
- the objective function $\varphi(y)$ satisfies the Lipschitz condition

$$\left|\varphi(y_1)-\varphi(y_2)\right| \le L\left\|y_1 - y_2\right\|,\ y_1, y_2 \in D$$

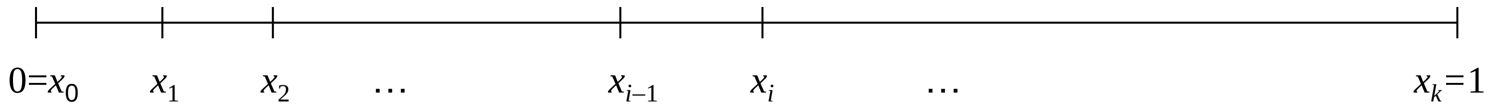
- the objective function $\varphi(y)$ may be “black box” function.



Globalizer Architecture



Algorithm of Global Search (AGS)



Let $x^0=0$, $x^1=1$.

1. For each (x_{i-1},x_i), $1\le i\le k$, calculate *characteristic* $R(i)$.
2. Find interval with maximum characteristic
   $R(t)=\max\{R(i)\colon 1\le i\le k\}$.
3. Make next trial at internal point of the interval $x^{k+1}\in(x_{t-1},x_t)$,
4. Check stop condition: $\Delta_t\le\varepsilon$

Algorithm of Global Search (AGS)

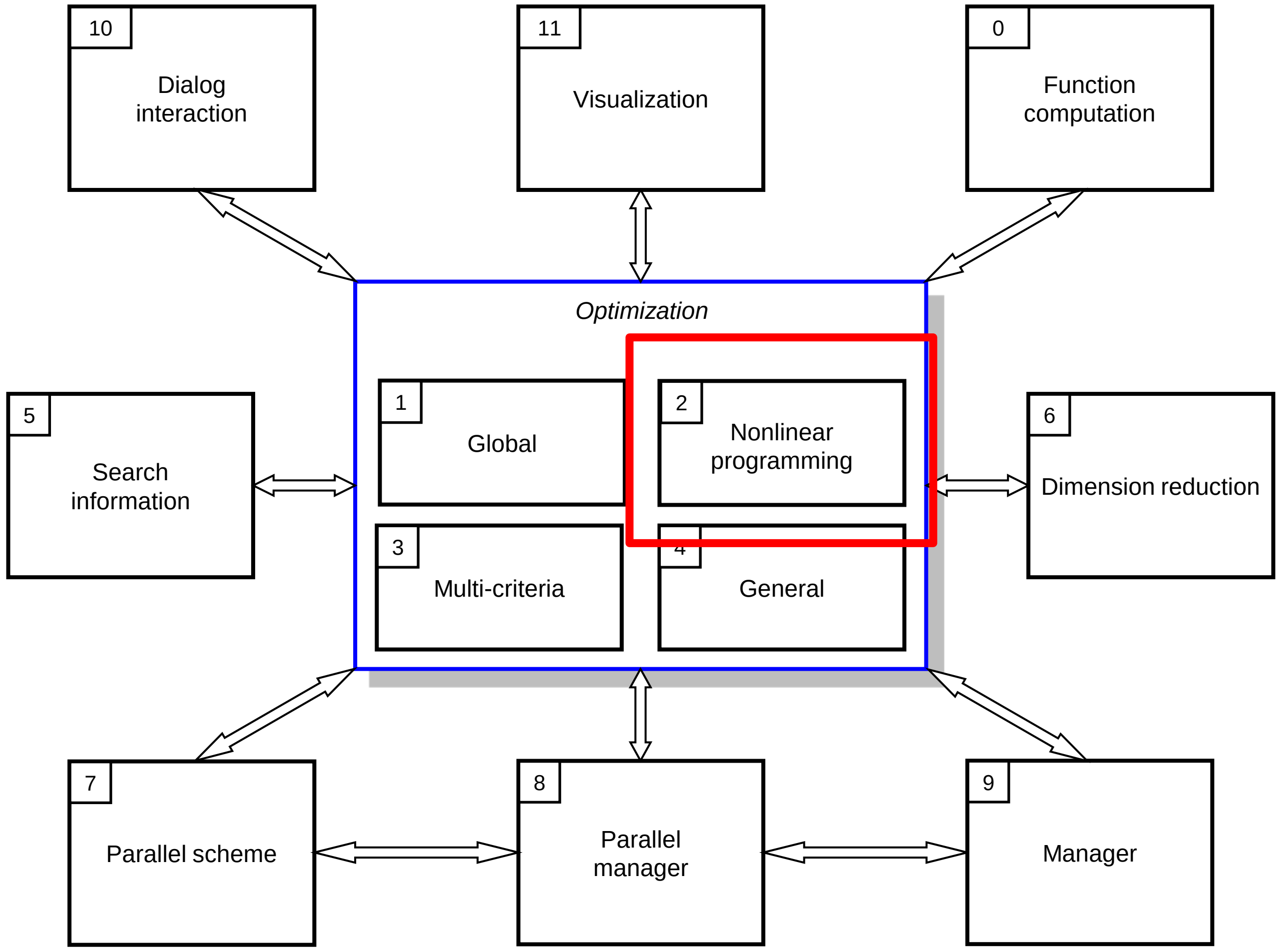
- Characteristic $R(i) = \Delta_i + \frac{(z_i - z_{i-1})^2}{r^2 \mu^2 \Delta_i} - 2\frac{(z_i + z_{i-1})}{r\,\mu},$

where $\mu = \max\left\{ \frac{|z_i - z_{i-1}|}{\Delta_i}, \quad i = 1,\dots,k \right\}$ is adaptive estimation of Lipschitz constant L, $r > 1$ – parameter.

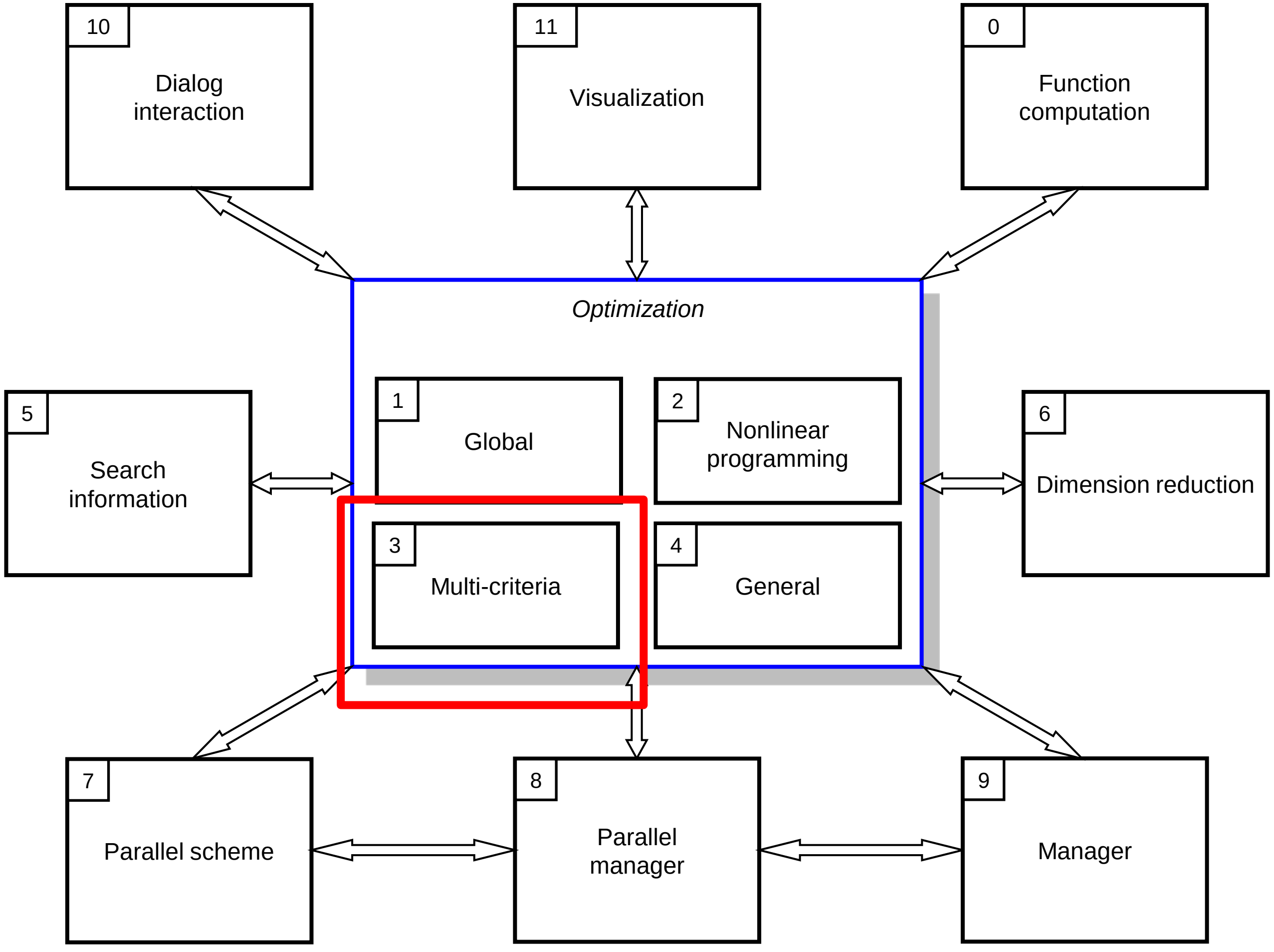
- New point $x^{k+1} = \frac{x_t + x_{t-1}}{2} - \text{sign}(z_t - z_{t-1})\frac{1}{2r}\left[\frac{|z_t - z_{t-1}|}{\mu}\right]^N$

Theory of convergence of AGS presented in [Strongin, Barkalov].

Globalizer Architecture



Globalizer Architecture



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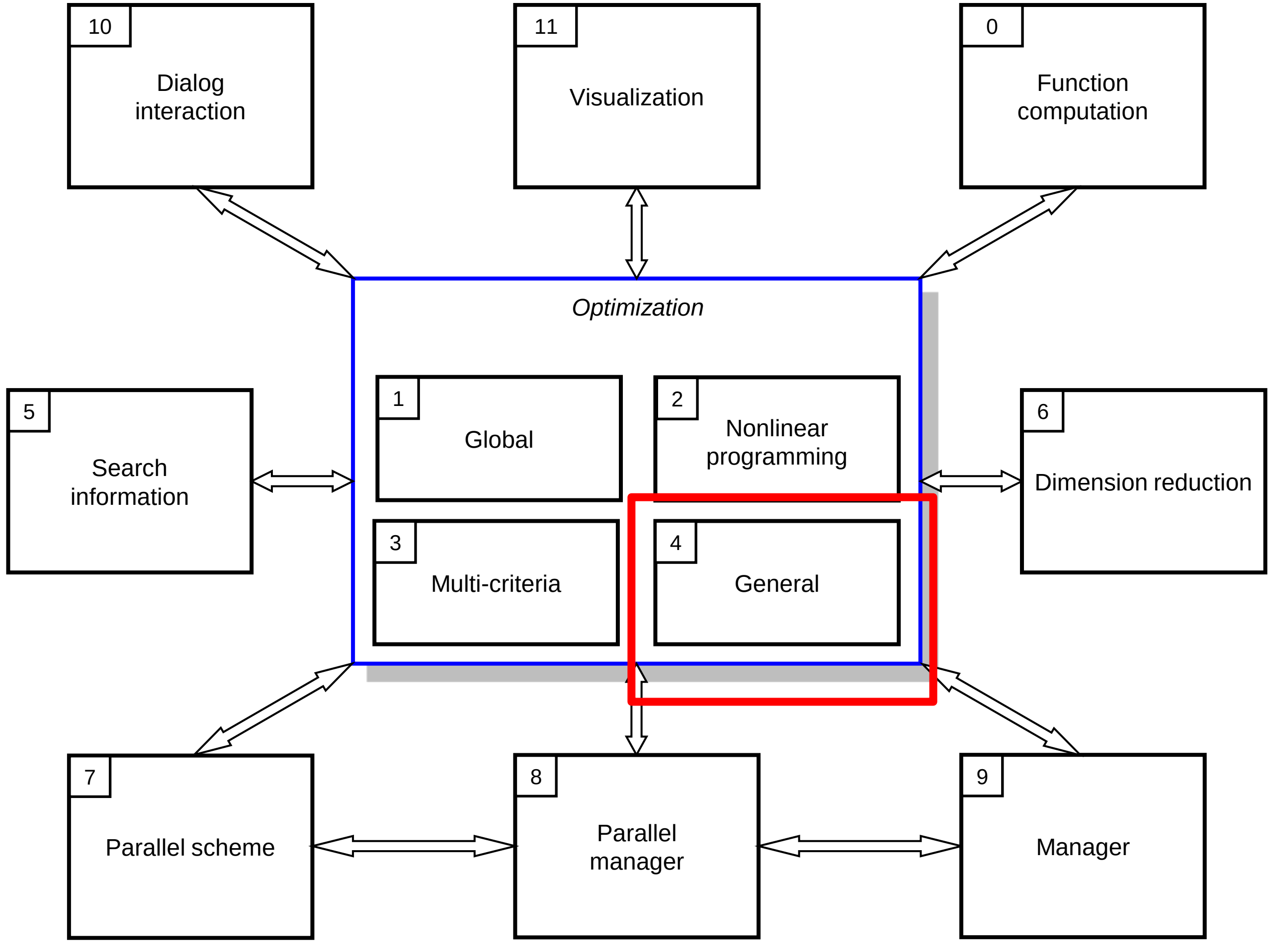
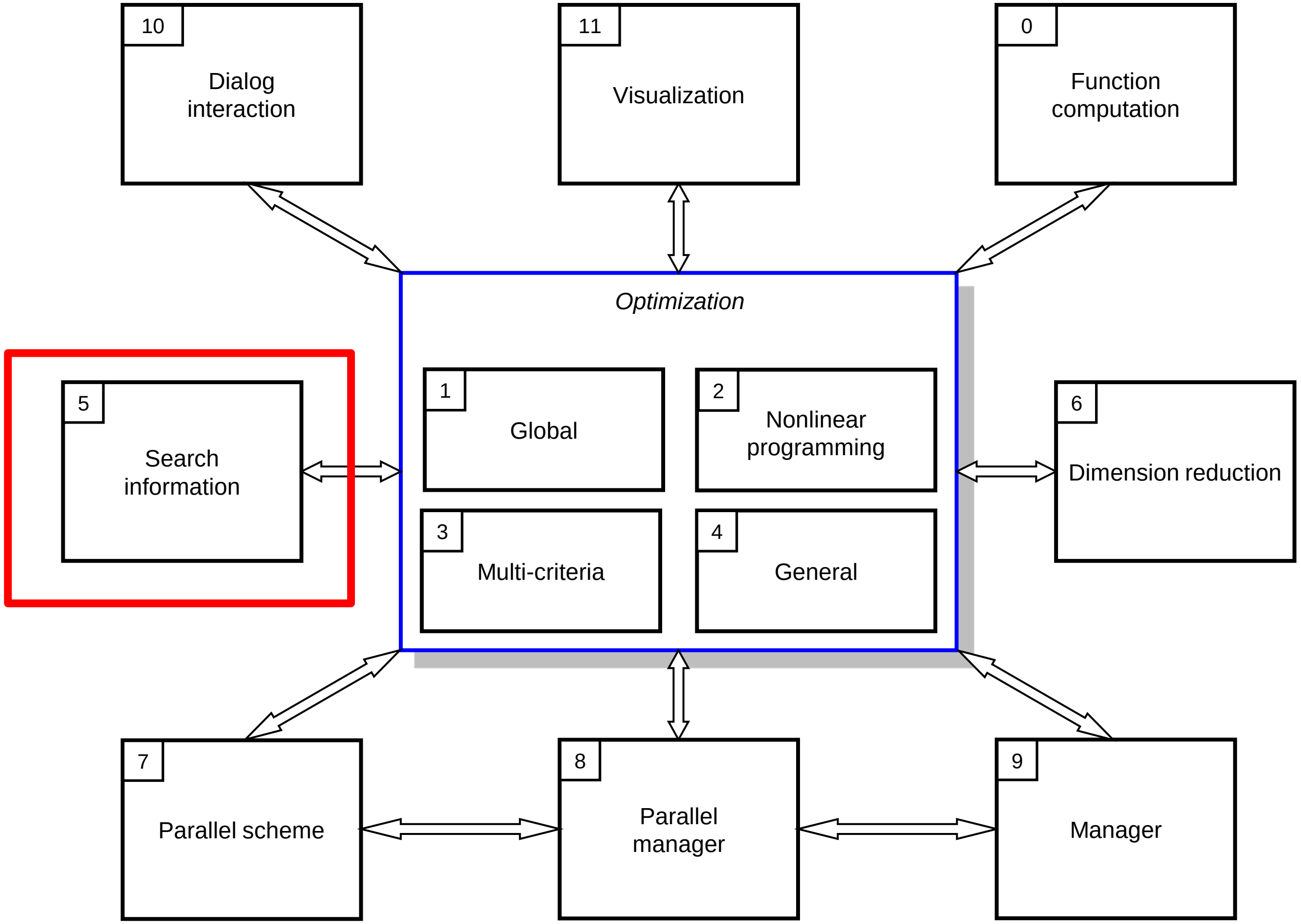
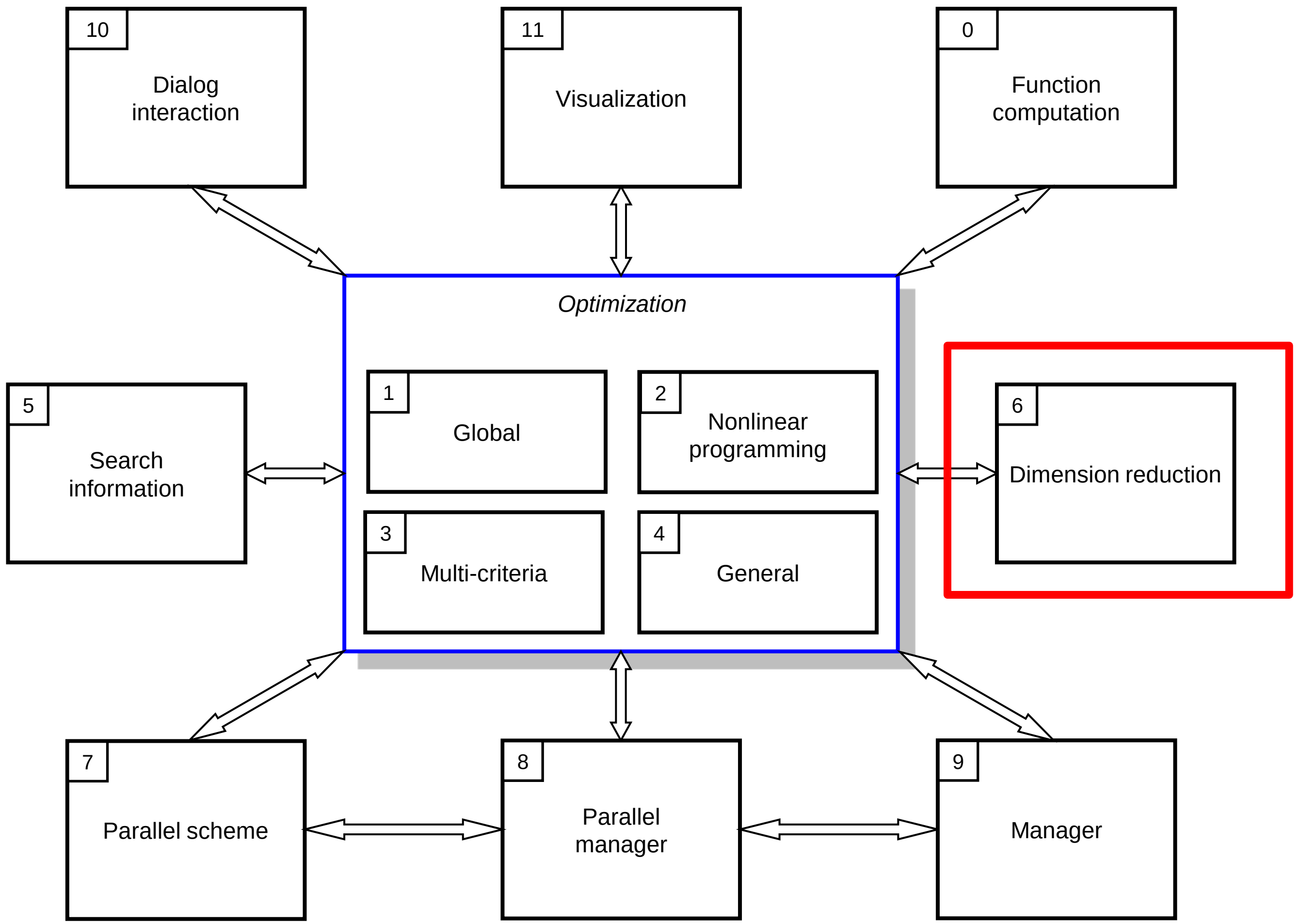


Globalizer Architecture



Globalizer Architecture



Reduction to one-dimensional problem

• The Globalizer implements a block multistage scheme of dimension reduction

• Initial vector y is represented as a vector of the «aggregated» macro-variables

$$y = (y_1, y_2, \dots, y_N) = (u_1, u_2, \dots, u_M)$$

where

$$u_1 = (y_1, y_2, \dots, y_{N_1})$$

$$u_2 = (y_{N_1+1}, y_{N_1+2}, \dots, y_{N_1+N_2})$$

$$u_M = (y_{N-N_M+1}, y_{N-N_M+2}, \dots, y_N)$$

and

$$N_1 + N_2 + \dots + N_M = N$$

Reduction to one-dimensional problem

- Using the macro-variables, the main relation of the well-known multistage scheme can be rewritten in the form

$$\min_{y\in D}\varphi(y) = \min_{u_1\in D_1}\min_{u_2\in D_2}\ldots\min_{u_M\in D_M}\varphi(y)$$

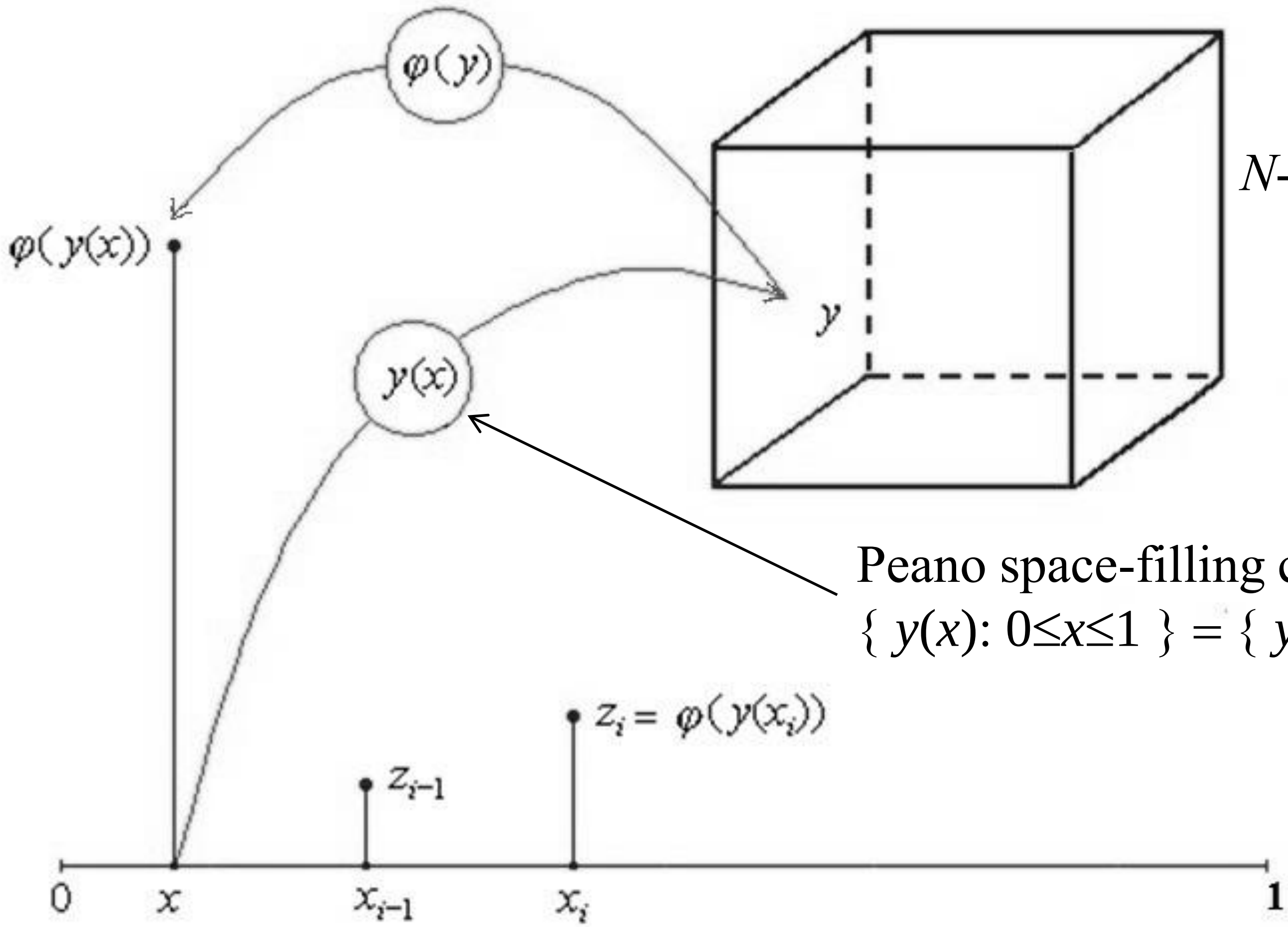
where D_i, $1 \le i \le M$, are the projections of D onto the subspaces corresponding to the macro-variables u_i, $1 \le i \le M$.

- The nested subproblems

$$\varphi_i(u_1,\ldots,u_i) = \min_{u_{i+1}\in D_{i+1}} \varphi_{i+1}(u_1,\ldots,u_i,u_{i+1}),\ 1 \le i \le M-1$$

are multidimensional and for their solution we can use the dimension reduction on the basis of the Peano curves

Reduction to one-dimensional problem



N-d space

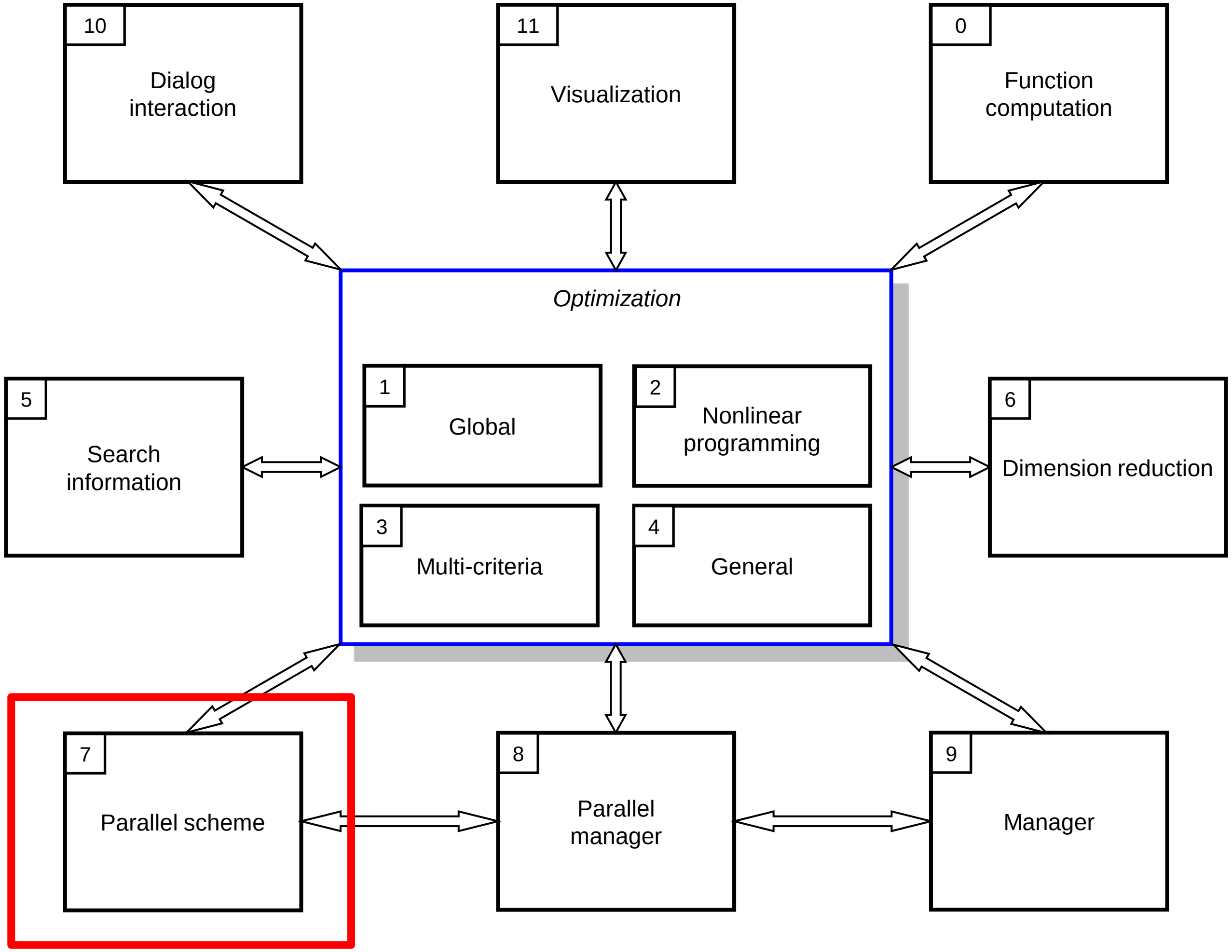
Peano space-filling curve

$\{ y(x): 0 \le x \le 1 \} = \{ y \in R^N: y \in D \}$

1-d scale

$$\varphi(y^*) = \min \{ \varphi(y): y \in D, g_j(y) \le 0, 1 \le j \le m \}$$
$$= \min \{ \varphi(y(x)): x \in [0,1], g_j(y(x)) \le 0, 1 \le j \le m \}$$

Globalizer Architecture



Parallel scheme

Options for parallelization:

- to carry out search domain decomposition;
- to parallelize calculation of the problem functions;
- to parallelize implementation of the algorithm computing rules for selection of the next trial point;
- **to change the algorithm for the purpose of carrying out several trials in parallel.**

Parallel scheme

- Consider a vector of parallelization degrees

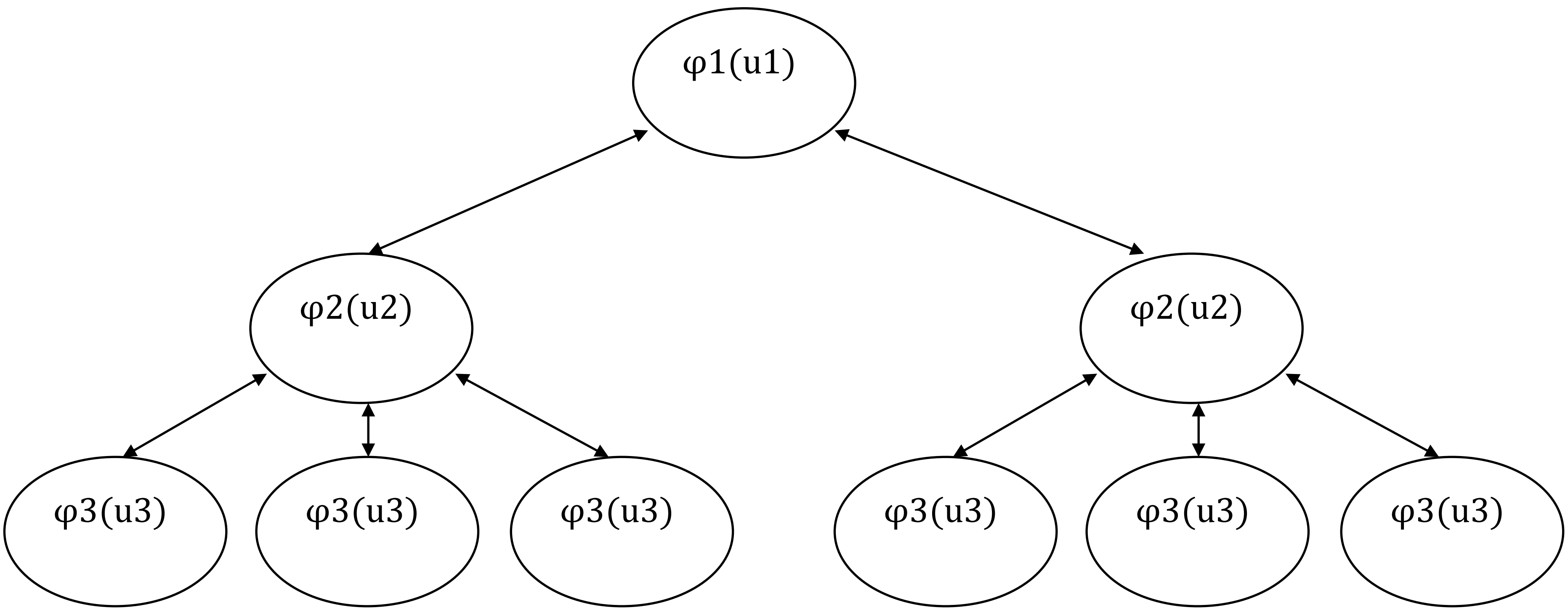
$$\pi = (\pi_1, \pi_2, \ldots, \pi_M)$$

- For the macro-variable u_i, the number π_i means the number of parallel trials at i-th level
- The total number of processors used will be

$$\Pi = 1 + \sum_{i=1}^{M-1} \prod_{j=1}^{i} \pi_j$$

Parallel scheme

- Process Tree (example, M = 3)



Parallel computing for distributed memory

Let's consider the *multiple mapping* $Y(x)$

$$Y(x)=\{y^1(x), y^2(x), \ldots, y^L(x)\}.$$

where $y_i(x)$ is transformed Peano curve.

Use of multiple mapping forms the set of L problems

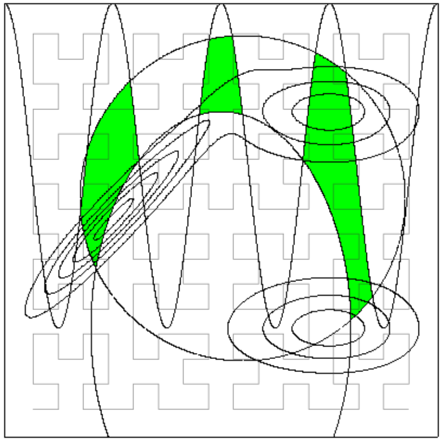
$$\min\{\varphi(y^l(x)): x\in[0,1], g_j(y^l(x))\le 0, 1\le j\le m\}, 1\le l\le L.$$

They can be solved in parallel.

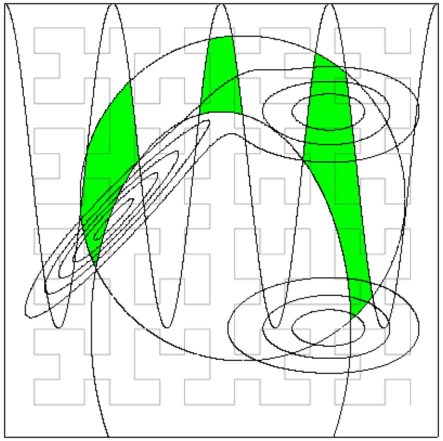
Each one-dimensional problem is solved on a separate processor/node.

Any computed value $z_i=g_\nu(y^l(x_i))$ for the problem l can be transformed to the value $z_j=g_\nu(y^k(x_j))$ for the problem k without time-consuming computing of the function $g_\nu(y)$.

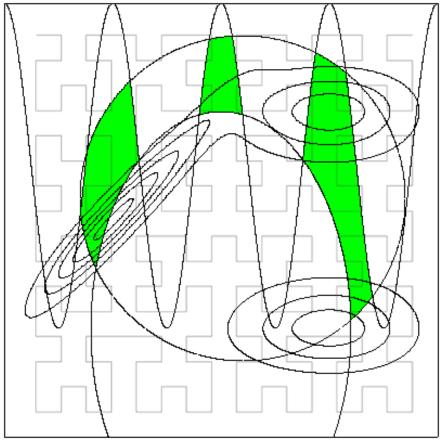
Parallel computing for distributed memory



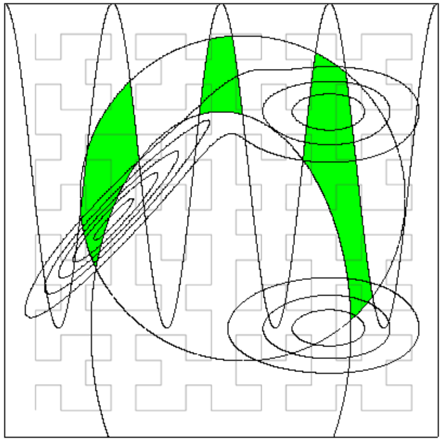
$\min\{\varphi(y^1(x)):x\in[0,1],\ g_j(y^1(x))\le 0,\ 1\le j\le m\}$



$\min\{\varphi(y^2(x)):x\in[0,1],\ g_j(y^2(x))\le 0,\ 1\le j\le m\}$

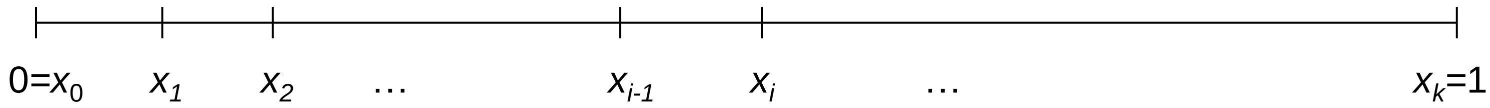


$\min\{\varphi(y^3(x)):x\in[0,1],\ g_j(y^3(x))\le 0,\ 1\le j\le m\}$



$\min\{\varphi(y^3(x)):x\in[0,1],\ g_j(y^3(x))\le 0,\ 1\le j\le m\}$

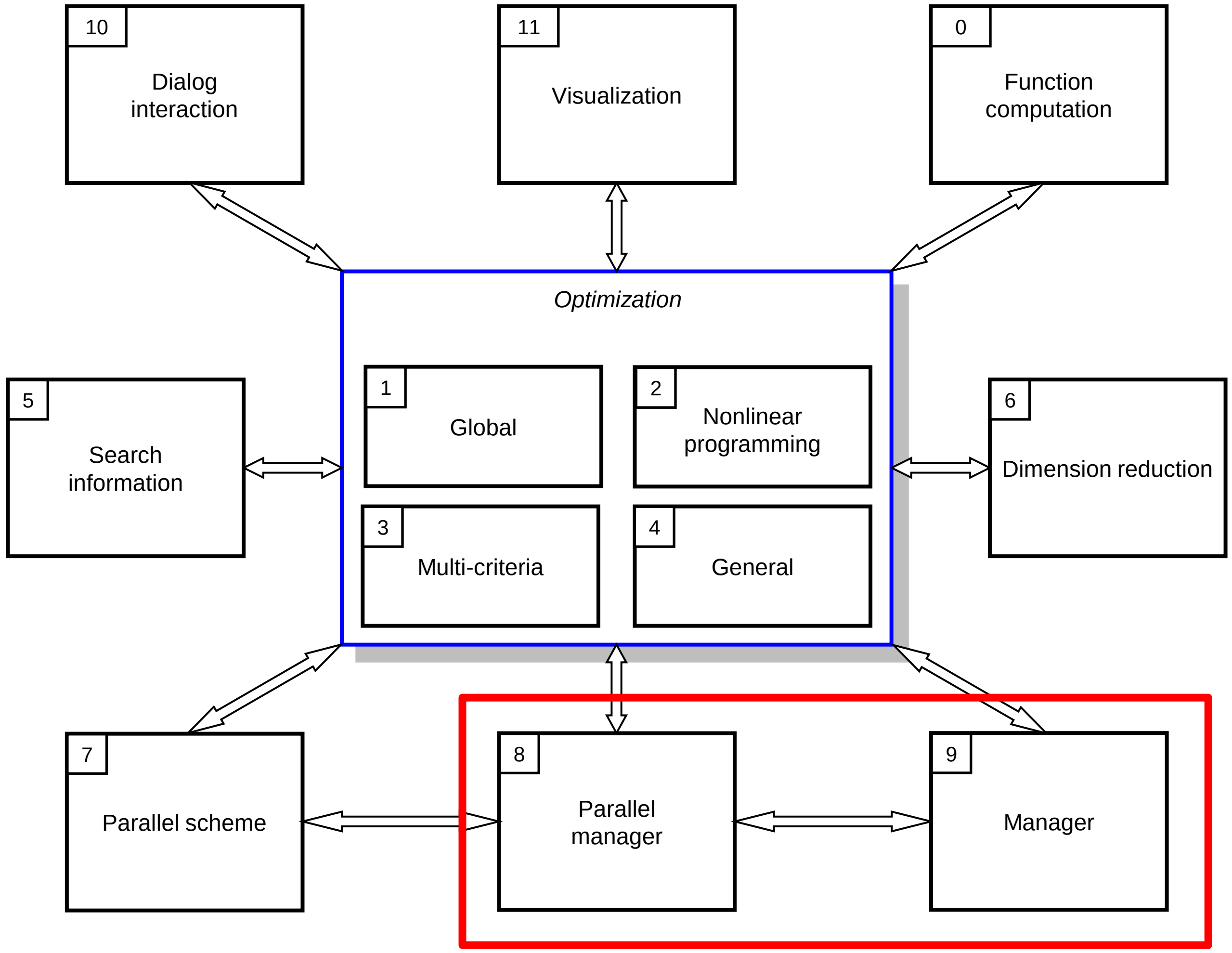
Parallel computing for shared memory



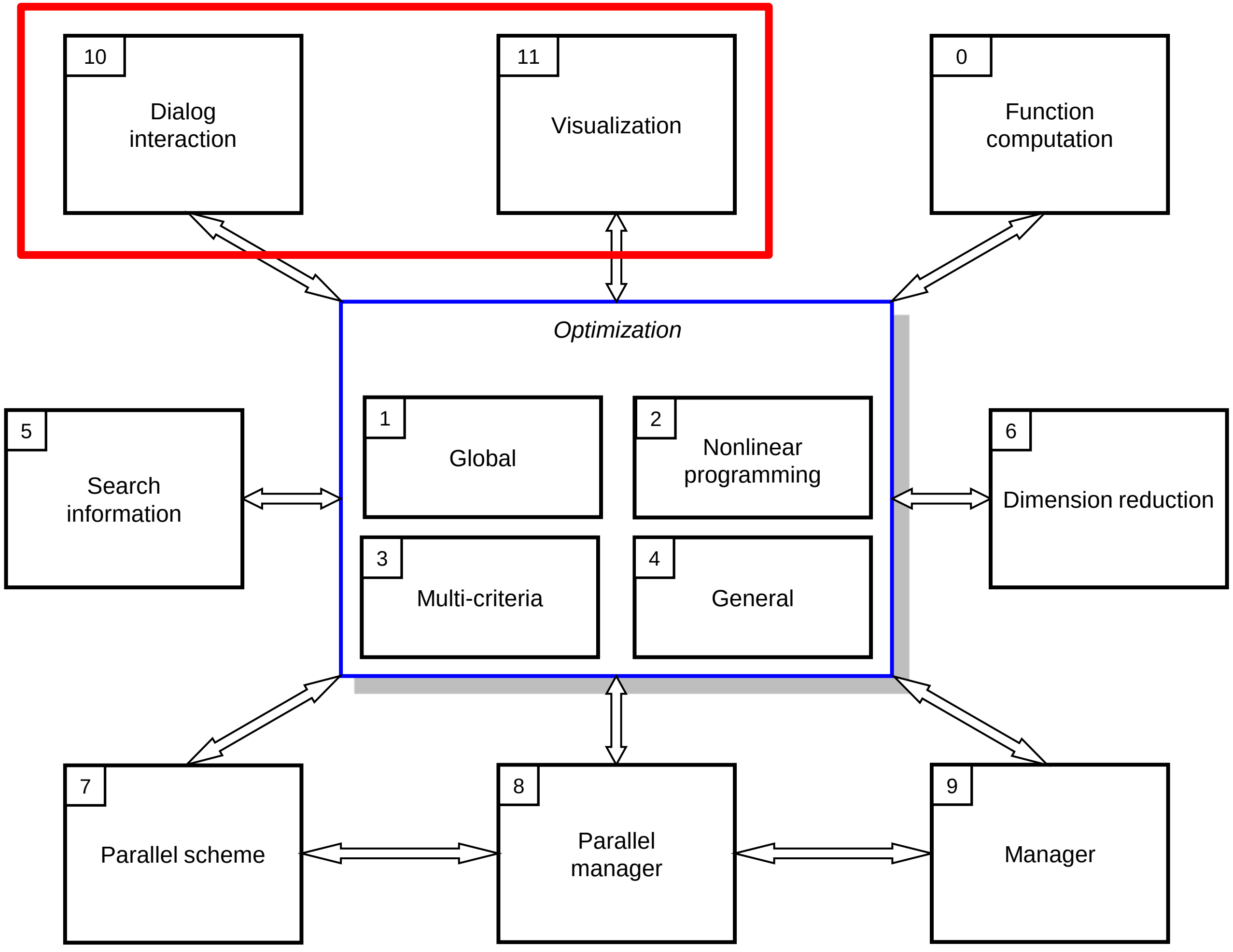
Let $x^0=0, \ldots, x^p=1$.

1. For each (x_{i-1}, x_i), $1 \le i \le k$, calculate *characteristic* $R(i)$.
2. **Sort intervals by characteristic, take p intervals with largest characteristics $R(t_1) > R(t_2) > \ldots > R(t_p)$**
3. **Make next p trials in parallel at internal points of the intervals (x_{t_1-1}, x_{t_1}), (x_{t_2-1}, x_{t_2}),… (x_{t_p-1}, x_{t_p})**
4. Check stop condition: $\Delta_{t_i} \le \varepsilon$

Globalizer Architecture



Globalizer Architecture



Numerical Results

- Vibration Isolation Problem

$$
\begin{aligned}
&\ddot{\xi}_1 = -\beta(\dot{\xi}_1 - \dot{\xi}_2) - \xi_1 + \xi_2 + u + v, \\
&\ddot{\xi}_2 = -\beta(\dot{\xi}_2 - \dot{\xi}_1) - \xi_2 + \xi_1 + v, \\
&\xi_1(0) = \xi_2\,(0) = 0, \qquad \dot{\xi}_1(0) = \dot{\xi}_2\,(0) = 0.
\end{aligned}
\tag{1}
$$

ξ_1 and ξ_2 are coordinates of the material points,

v is the base acceleration up to sign (the external excitation),

u is the control force,

β is a positive damping parameter.

Numerical Results

- Let's rewrite (1) in the standard form

$$
\begin{aligned}
&\dot{x_1} = x_3, \\
&\dot{x_2} = x_4, \\
&\dot{x_3} = -x_1 + x_2 - \beta x_3 + \beta x_4 + v + u, \qquad\qquad (2) \\
&\dot{x_4} = x_1 - x_2 + \beta x_3 - \beta x_4 + v, \\
&x_1(0) = x_2(0) = x_3\,(0) = x_4\,(0) = 0.
\end{aligned}
$$

- This model can describe the typical situations of vibration isolation for devices, apparatuses and humans located on moving vehicles.

Numerical Results

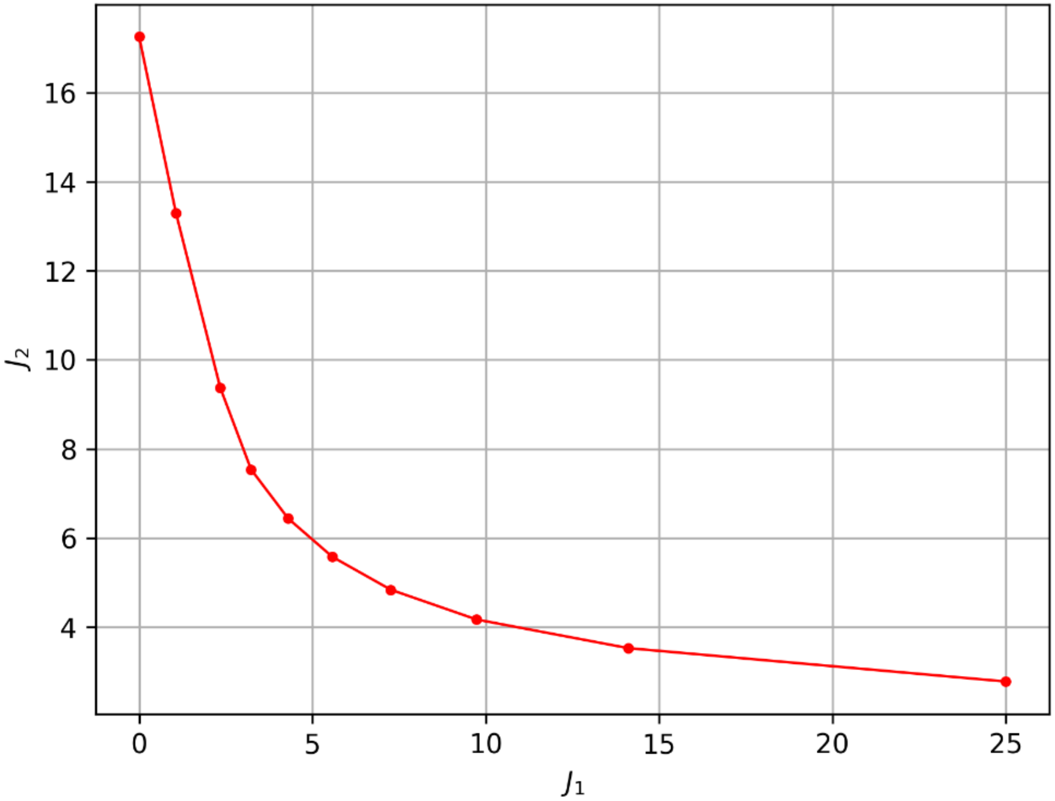
- Choose two criteria for this system to characterize the process of vibration isolation

$$J_1(u) = \sup_{v \in L_2} \frac{\sup_{t \geq 0} |x_1(t)|}{\|v\|_2},$$

$$J_2(u) = \sup_{v \in L_2} \frac{\sup_{t \geq 0} |x_2(t) - x_1(t)|}{\|v\|_2}.$$

Numerical Results

- Consider two-objective control problem for state-feedback case.
- The Pareto optimal front computed by Globalizer



Thank you for attention

References

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