Dynamic optimization of linear solver parameters in modelling of unsteady filtration processes

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Contents

Linear systems and linear solvers

- Solution of linear systems
- Linear systems are differ
- A lot of linear solvers
- A lot of parameters for each linear solver
- How to choose the linear solver
- How to choose the linear solver parameters

Goal

- Time optimization of $A_k x = b_k$
- INMOST
- BIILU2 linear solver
- τ - threshold parameter
- q - overlap parameter

Properties

$$T_k = \phi(A_k, b_k, p, \varepsilon_k) \equiv \Phi(A_k, b_k, p) \pm \varepsilon_k \equiv Time(A_k x = b_k)$$

- T_k may differ even for the same A_k, b_k, p
- The value of ε_k is imposible to predict
- T_k can be calculated only once
- Optimal value of p may depend on k
- $min T_k$ may encrease during simulation

Several approaches

- Prescribed optimal fixed parameters
- Brute-force search (direct search over all set of parameters)
- Fast simulated re-annealing optimization algorithm
- Alternating parameters probe based tuning (1U)

Fast simulated re-annealing

- Effective random search
- Local terms: “temperature”, “energy”, probability of jump
- Previous values of T_k are not required
- Does not “stop” at the local minima

Alternating parameters probe based tuning (1U)

- Find local minima
- Check nearby area
- Move to global minima

Models

- Steady
  - $A_k \equiv A$ and $b_k \equiv b$
  - Research the impact of the parameter p on solution time
- Model function
  - Easy to debug and to test
  - Optimal parameter p depends on "time"
- Unsteady
  - Black-Oil simulator BOSS
  - Optimal parameter p depends on "time"
  - $minT_k$ depends on simulation step k

INM cluster configuration

- Compute Node Arbyte Alkazar+ R2Q50;
- 16 cores (two 8-core processors Intel Xeon E5-2665@2.40GHz);
- 64 Gb RAM;
- SUSE Linux Enterprise Server 11 SP1 (x86_64).

N14 sample problem

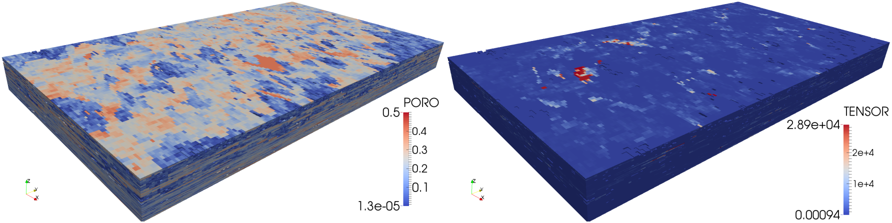

Figure : The porosity and permeability distributions for SPE-10 problem

Black-Oil Simulator for Scholars (BOSS) for SPE-10 problem.
The size of the model mesh is $60 \times 220 \times 85$ cells ($1.122 \cdot 10^6$ cells).
The porosity varies from $1.3 \cdot 10^{-5}$ to 0.5 (see Fig. left).
The permeability varies from 10^{-3} to $3 \cdot 10^4$ (see Fig. right).
The model has 5 vertical wells completed throughout formation.
The dimension of the linear system N14 is 3 896 013 unknowns.

N14

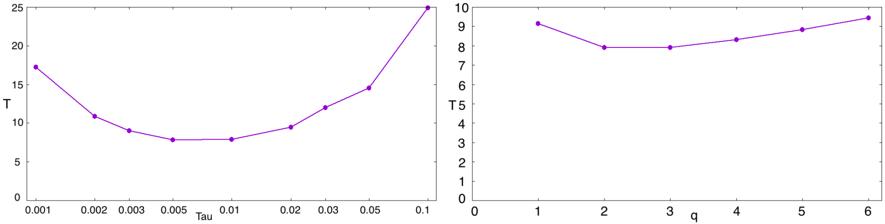

Figure : Total solution time T in s. for N14 depending on τ and q for $p = 16$

$T = f(q = 3, \tau)$ and $T = f(q, \tau = 0.003)$, respectively.

N14

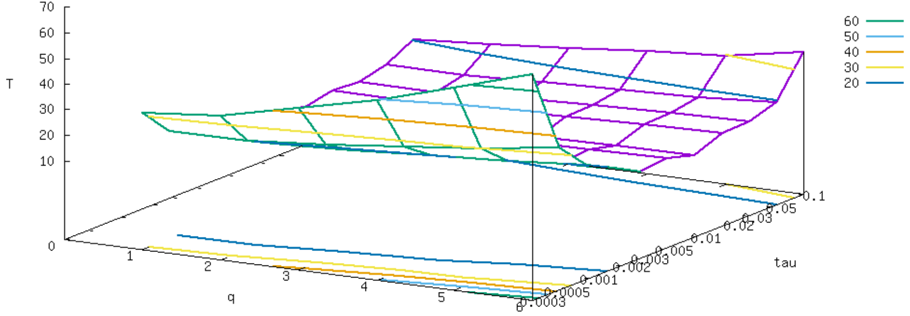

Figure : Total solution time T in s. for N14 in variables τ and q for $p = 16$

$f(\tau, q)$ – special function instead of real solution time

$$f(\tau, q) = \left(\frac{16}{25}(\lg(\tau/\tau_0))^2 + 1\right)\left(\frac{1}{25}\left(\frac{17.5(q - q_0)}{7.5 + q - q_0}\right)^2 + 1\right)$$

$$\tau_0 = 0.003, \quad q_0 = 3$$

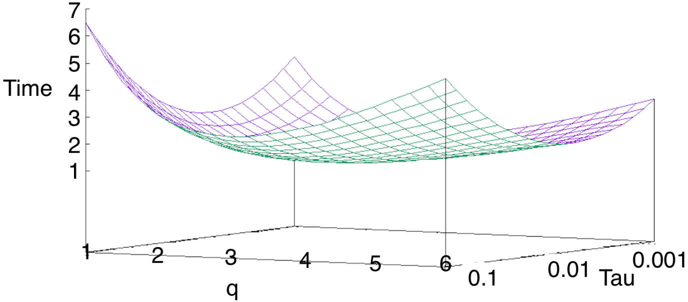

Figure : Two-parameter function $f(\tau, q)$

$f(\tau, q)$

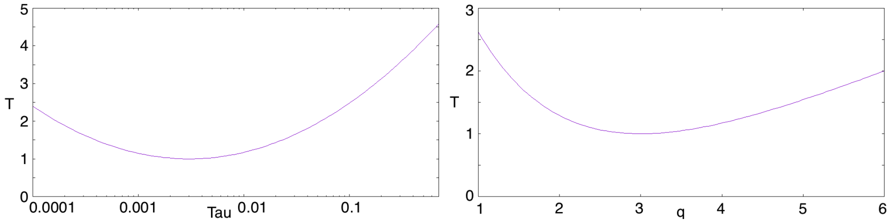

Figure : Cross-sections for $q = q_{\rm opt} = 3$ and $\tau = \tau_{\rm opt} = 0.003$

$T = f(q = 3, \tau)$ and $T = f(q, \tau = 0.003)$, respectively.

$f(\tau, q, t)$ – unsteady

$$\tau_0 = 10^{-2-\cos(2\pi t/t_0)}$$

$$q_0 = 2 + \cos(2\pi t/t_0)$$

$$t_0 = 100$$

Here, $\lg(\tau) \in [-3; -1]$ and $q \in [1; 3]$.

$f(\tau, q, t)$ – unsteady

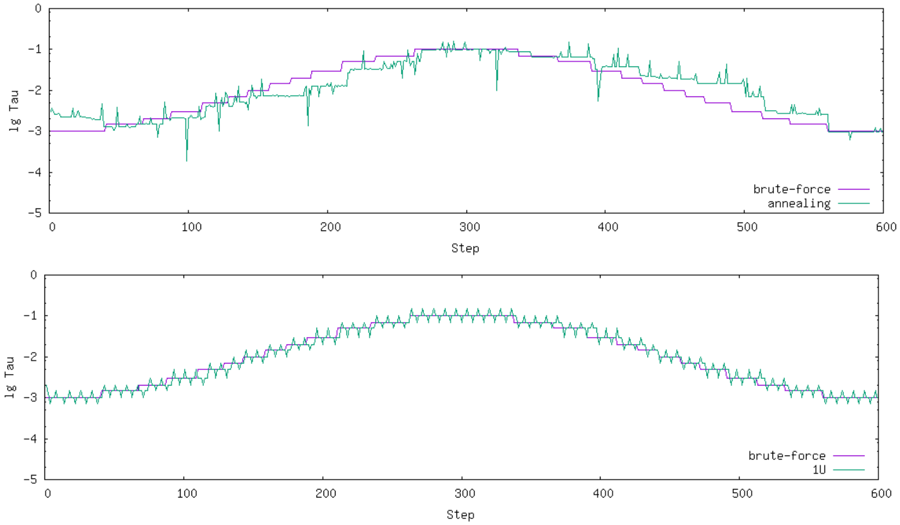

Figure : τ_{opt} depending on the time step k for function $f(\tau, q, t)$

Brute-force search vrs. SA algorithm and 1U algorithm, respectively.

Unsteady black-oil simulation – fixed parameters (τ, q)

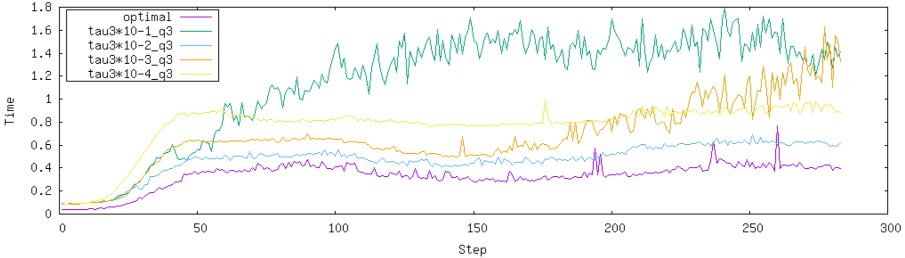

Figure : Unsteady black-oil simulation solution times depending on time step k

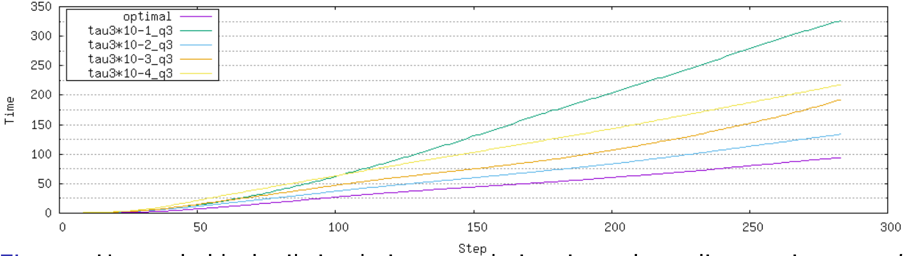

Figure : Unsteady black-oil simulation cumulative times depending on time step k

Unsteady black-oil simulation – parameters optimization

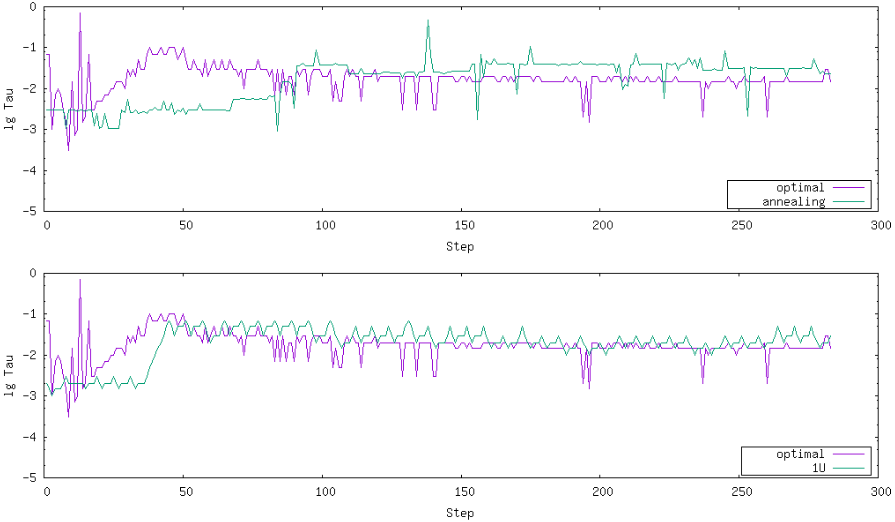

Figure : Optimizing τ for black-oil simulator

Unsteady black-oil simulation – parameters optimization

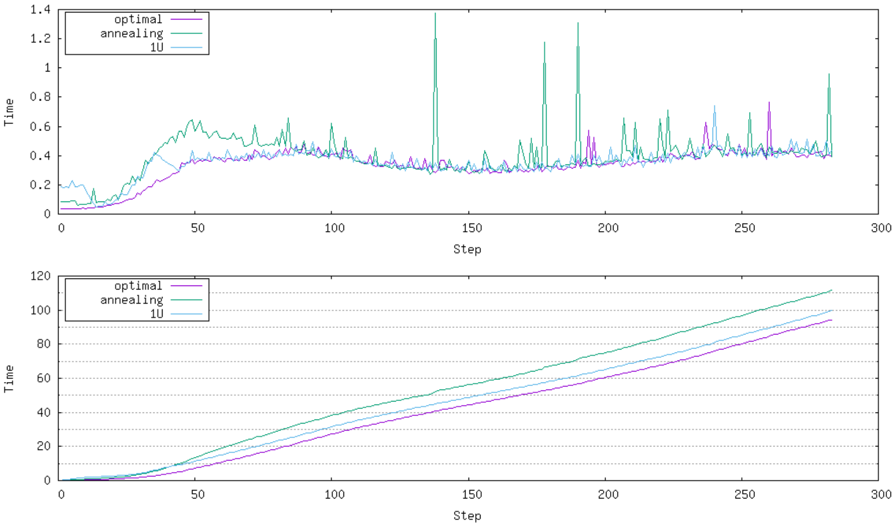

Figure : Local and cumulative times depending on time step k

Unsteady black-oil simulation – parameters optimization

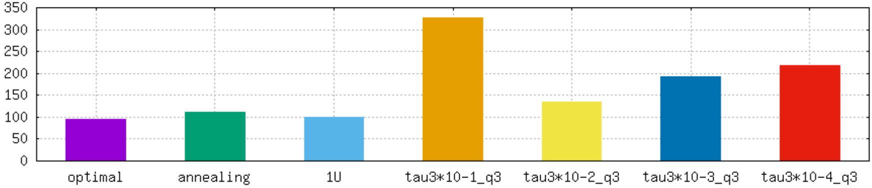

Figure : Cumulative times bar chart for default sets of parameters and for proposed algorithms compared with the optimal one

Conclusion

- Two parameters optimization algorithms are proposed
- The solution time is close ($\approx$10%) to the optimal one
- Better than any prescribed set of parameters
  - 2-3-4 times better than regular set
  - 1.5 times better than the best fixed one

- To be applied to:
  - another linear solvers like PETSc AS(q)+ILU(k)
  - another application, i.e. haemodynamics