Supercomputer modelling of electromagnetic wave scattering with boundary integral equation method

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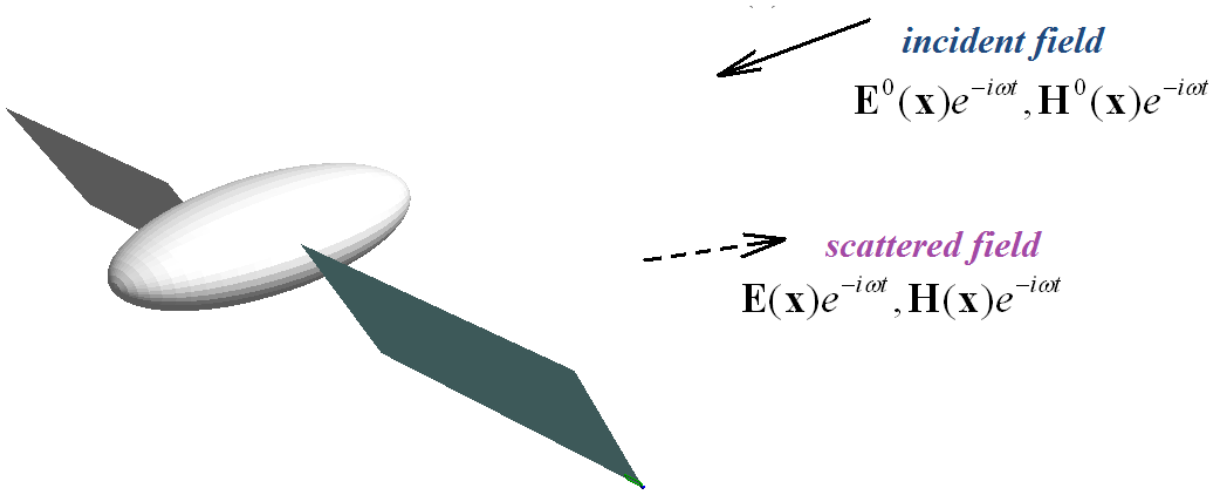
STATEMENT OF THE PROBLEM



Ω - domain outside bodies, Σ - total surface of bodies and screens

incident field

$\vec{\mathbf{E}}^0(x)e^{-i\omega t}, \vec{\mathbf{H}}^0(x)e^{-i\omega t}$

$\vec{\mathbf{E}}^0(x) = \vec{\mathbf{E}}_0 e^{i\vec{\mathbf{k}}\vec{\mathbf{r}}},$

$\vec{\mathbf{H}}^0(x) = \dfrac{e^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}}{\omega\mu\mu_0}\vec{\mathbf{k}} \times \vec{\mathbf{H}}_0,$

$\vec{\mathbf{k}} = k\vec{\mathbf{k}}_0$ - wave vector,

$\left|\vec{\mathbf{k}}_0\right| = 1, k^2 = \omega^2\varepsilon\varepsilon_0\mu\mu_0,$

$\vec{\mathbf{r}}$ - radius vector of a point x

Maxwell's equations: $rot\,\mathbf{E} = i\mu\mu_0\omega\mathbf{H},\ rot\,\mathbf{H} = -i\varepsilon\varepsilon_0\omega\mathbf{E}$

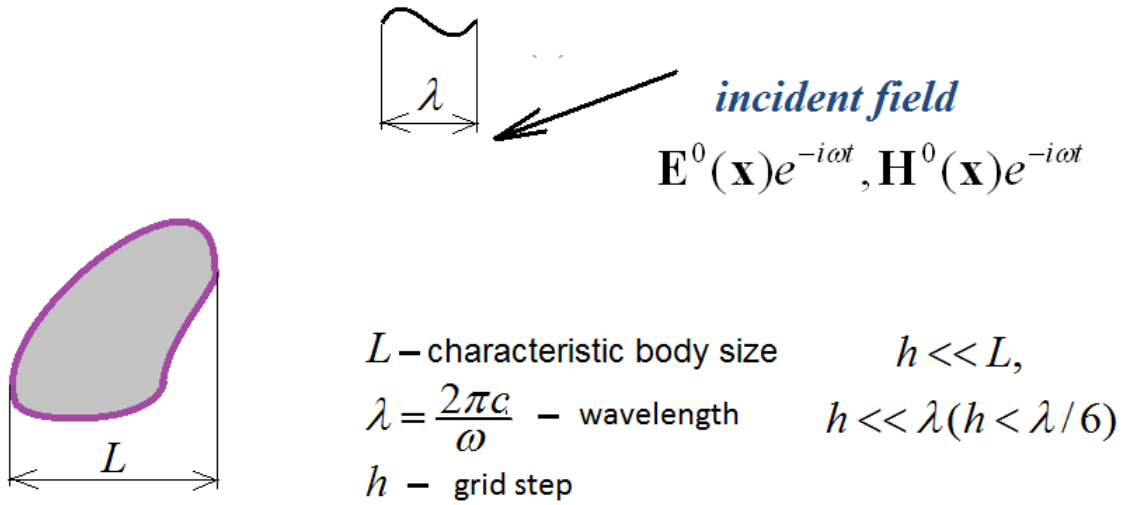
Boundary conditions: $\mathbf{E}_{full} \times \mathbf{n} = 0$ on a perfectly conducting surfaces of bodies and screens

Energy finiteness condition: $\mathbf{E} \in L_2^{loc}(\Omega),\ \mathbf{H} \in L_2^{loc}(\Omega)$

Sommerfeld radiation conditions at infinity:

$$\frac{\mathbf{d}}{\mathbf{d}\tau}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix} - ik\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix} = o\left(\frac{1}{|\mathbf{x}|}\right), \quad \boldsymbol{\tau} = \frac{\mathbf{x}}{|\mathbf{x}|}, \text{ при } |\mathbf{x}| \to \infty.$$

COMPUTATIONAL COMPLEXITY OF PROBLEMS



Grid and finite-element methods based on discretization of electromagnetic field in space:
- to fulfil boundary conditions on infinity one have to use calculating domain many times exceeding size of bodies
- the requirement of smallness of discretization step in comparison to wave length puts the limitation on utilisation of non-uniform meshes.

Boundary integral equations methods:
- the grid is built only on the surface of the bodies
- modelling of monochromatic wave processes in homogeneous environment

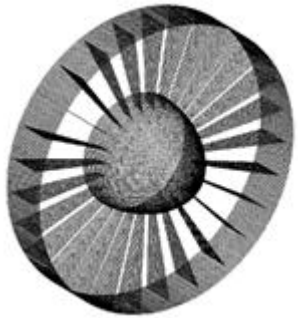
Boundary integral equations methods:

- $\lambda << L$ - **The physical theory of diffraction is applicable (Analytic approximations of solutions of integral equations exist)**
- $\lambda \sim L$, $\lambda > L$ - **it is necessary to solve integral equations**

most difficult case

- $\lambda \approx L/10$,

- large object with a large number of small parts



integral representation of an electric field

$$\mathbf{E}(x) = grad\, div \iint\limits_{\Sigma} \mathbf{j}(y) F(x-y) d\sigma_y + k^2 \iint\limits_{\Sigma} \mathbf{j}(y) F(x-y) d\sigma_y$$

$$F(x-y) = \frac{e^{ikR}}{R}, \quad R = |x-y|, k = \frac{\omega}{c}, c = \frac{1}{\sqrt{\varepsilon\mu}}.$$

integral equation:

$$\int_{\Sigma} \mathbf{n}(x) \times \mathbf{K}(\mathbf{j}, x, y) d\sigma_y = \mathbf{f}(x),\ x \in \Sigma,\ \mathbf{f}(x) = -\mathbf{n}(x) \times \vec{\mathbf{E}}^0(x)$$

core of the integral equation:

$$\mathbf{K}(\mathbf{j}, x, y) = grad_{\mathbf{x}}\, div_{\mathbf{x}} \left(\mathbf{j} F(x-y) \right) + k^2 \mathbf{j} F(x-y),\ \mathbf{j}, x, y \in R^3$$

$$\mathbf{K}(\mathbf{j}, x, y) = \mathbf{K}_0(\mathbf{j}, x, y) + \mathbf{K}_1(\mathbf{j}, x, y)$$

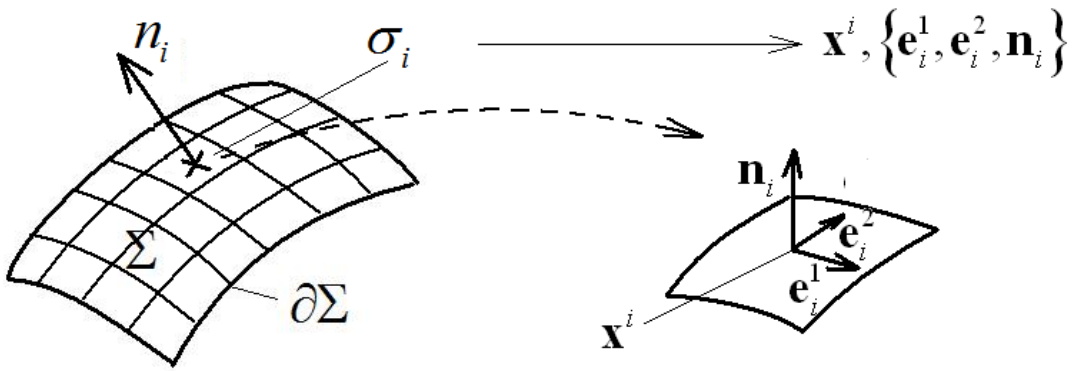
$$\mathbf{K}_0(\mathbf{j}, x, y) = \frac{-\mathbf{j} + 3\mathbf{r}(\mathbf{r}, \mathbf{j})}{R^3},$$

$$\mathbf{K}_1(\mathbf{j}, x, y) = \left(\mathbf{j} - 3\mathbf{r}(\mathbf{r}, \mathbf{j}) \right) \frac{1 - e^{ikR} + ikRe^{ikR}}{R^3} + \left(\mathbf{j} - \mathbf{r}(\mathbf{r}, \mathbf{j}) \right) \frac{k^2 e^{ikR}}{R},\ \mathbf{r} = \frac{x-y}{R},$$

$$|\mathbf{K}_0(\mathbf{j}, x, y)| \le O\left(\frac{1}{R^3} \right),\ |\mathbf{K}_1(\mathbf{j}, x, y)| \le O\left(\frac{1}{R} \right)$$

Integral is understood as hyper singular in the sense of the Hadamard finite value

THE NUMERICAL SCHEME



$\mathbf{j}_i = \mathbf{j}(\mathbf{x}^i)$ **-** current on cell number i

$\mathbf{j}_i^*(y) = (\mathbf{n}_i \times \mathbf{j}_i) \times \mathbf{n}(y)$ - current approximation on a cell

system of operator equations

$$\sum_{j=1}^{n} A_{ij}\mathbf{j}_j = \mathbf{f}_i, i = 1,\ldots,n$$

$$A_{ij}\mathbf{j}_j = \mathbf{n}(x_i) \times \int_{\sigma_j} \mathbf{K}(\mathbf{j}_j^*(x_i), x_i, y) d\sigma_y$$

equations for the coordinates of currents in local bases

$$\mathbf{j}_i = j_1^i \mathbf{e}_i^1 + j_2^i \mathbf{e}_i^2$$

$$\sum_{\substack{j=1,\ldots,n \\ l=1,2}} a_{ij}^{kl} j_j^l = f_i^k, i = 1,\ldots,n, k = 1,2$$

$$a_{ij}^{kl} = \left(A_{i,j}\mathbf{e}_j^l, \mathbf{e}_i^k\right), \; f_i^k = \left(\mathbf{f}_i, \mathbf{e}_i^k\right)$$

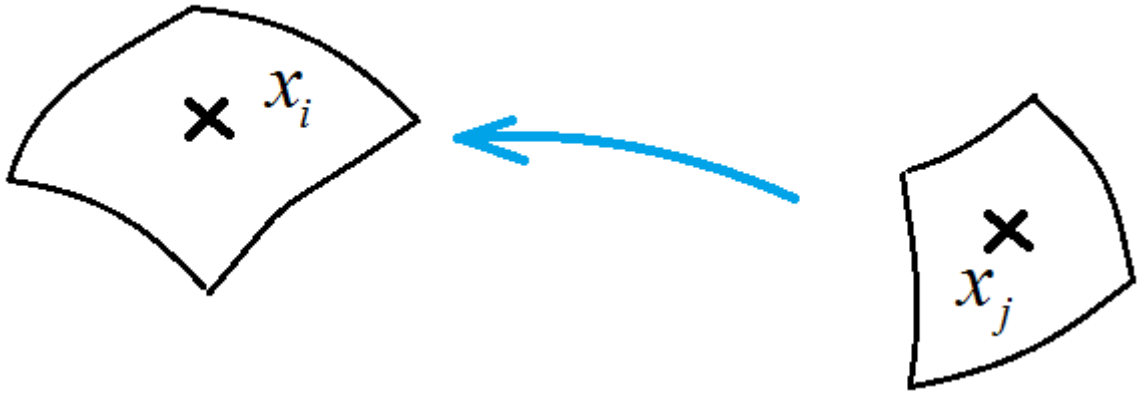
ON THE SOLUTION OF A SYSTEM OF LINEAR EQUATIONS

1) the matrix has a block structure $A_{ij} = \begin{pmatrix} a_{ij}^{11} & a_{ij}^{12} \\ a_{ij}^{21} & a_{ij}^{22} \end{pmatrix}$ -

2) the matrix is filled

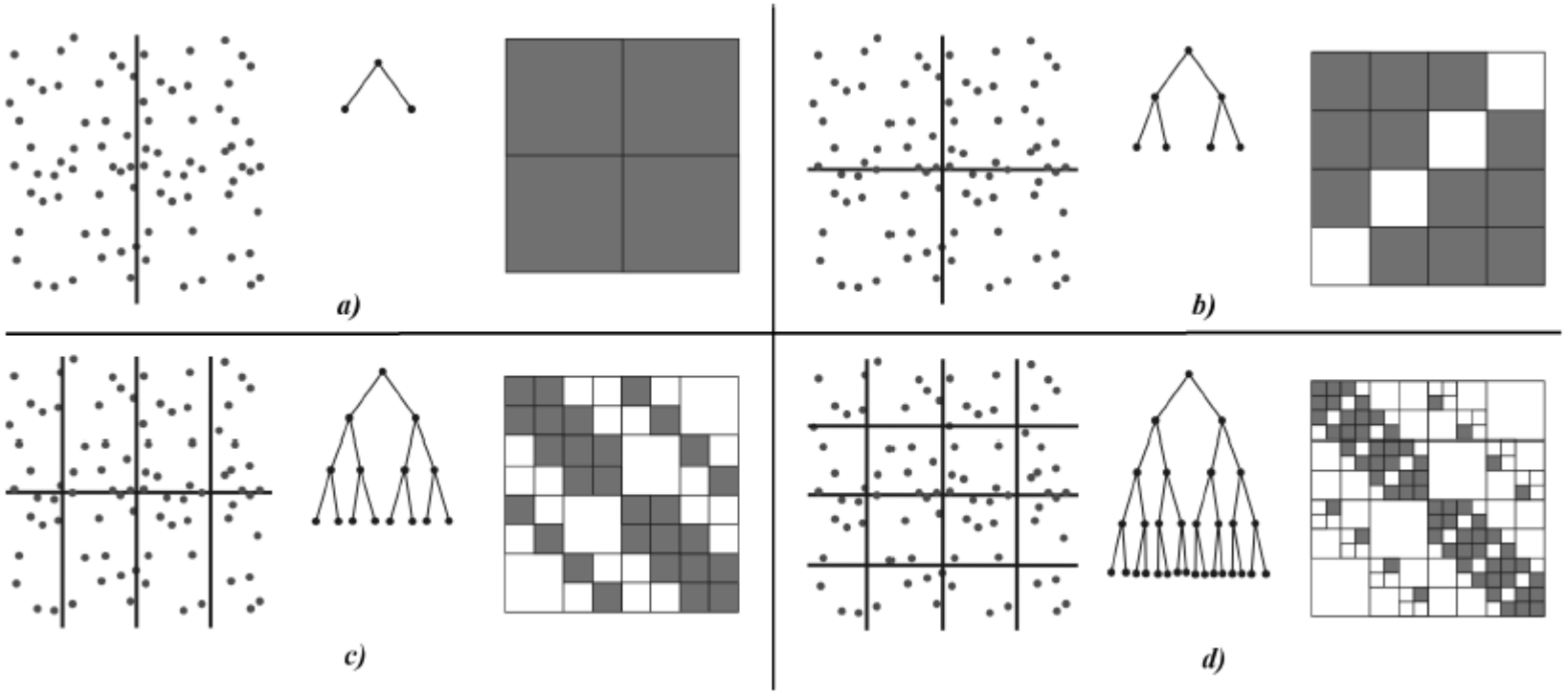
3) A_{ij} - the block describes the interaction of the source cell with the number i and the cell of the receiver with the number j.
We can talk about the interaction of the source point x_j and the point of the receiver x_i

4) Interaction decreases with increasing distance between the source and receiver



MOSAIC-SKELETON APPROXIMATION

Goreinov S.A., Tyrtyshnikov E.E., Zamarashkin N.L. A theory pseudo-skeleton approximations. // Lin. Algebra Appl. 261:1-21, 1997.



1) We separate points (receivers and sources) into clusters
2) Each pair of clusters corresponds to some block of the matrix
3) Each block is represented in the form $A = U \times V$,

$$\underset{m' \times n'}{\mathbf{A}} = \underset{m' \times r}{\mathbf{U}} \; \underset{r \times n'}{\mathbf{V}} \qquad r << m', n'$$

IMPROVING COMPUTING PERFORMANCE

- using standard Scalapack procedures for LU factorization or with GMRES iterative method.

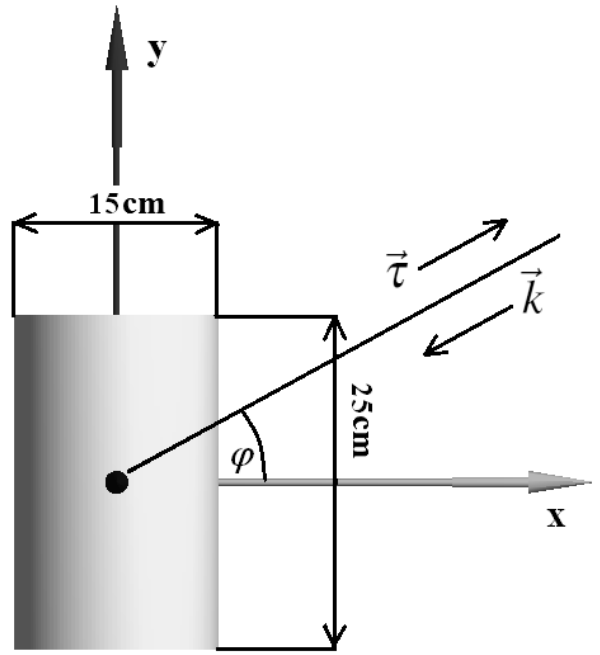
- using Mosaic-skeleton approximation method

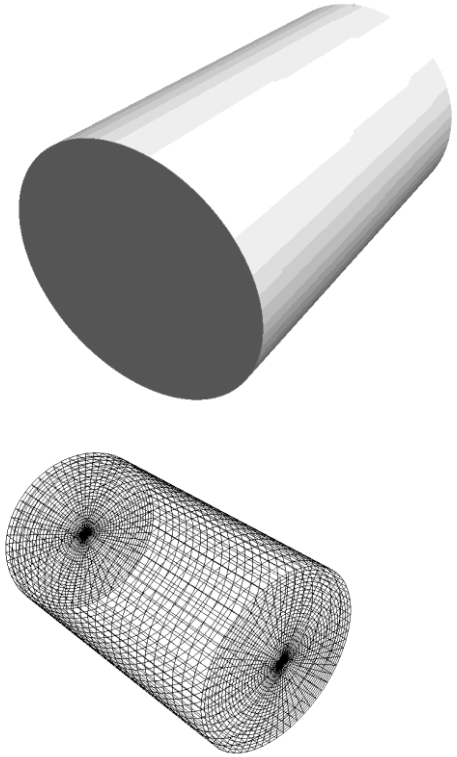
- using of parallel implementation of the Mosaic-skeleton method

A critical problem is the large RAM memory needs for the matrix storage

When using the mosaic-skeleton method, the degree of compression of the matrix depends on the particular problem

SCATTERING OF AN ELECTROMAGNETIC WAVE BY CYLINDER





Height of cylinder - $H = 25cm$

Radiation frequency

4 GHc - $\lambda = 7.5cm = 0.3H$

8 GHc - $\lambda = 3.75cm = 0.15H$

16 GHc - $\lambda = 1.875cm = 0.075H$

Radar cross section (RSC)

$$\sigma(\boldsymbol{\tau}) = \lim_{R\to\infty} 4\pi R^2 \frac{|\mathbf{E}(R\boldsymbol{\tau})|^2}{|\mathbf{E}_{ent}|^2}$$

THE EFFICIENCY OF THE COMPUTATIONAL ALGORITHM

Table 1. Required storage for matrix.

n	4GHz	8GHz	16GHz	full matrix
21760	1.962Gb (7.0%)	2.340Gb (8.3%)	2.999Gb (11%)	28.223Gb
45784	4.457Gb (3.6%)	5.369Gb (4.3%)	6.927Gb (5.6%)	124.920Gb
273600	—	—	48.998Gb (4.39%)	1115.456Gb

Table 2. Acceleration of matrix calculations for various number of processors. Number of cells 273600. Frequency 16 GHz.

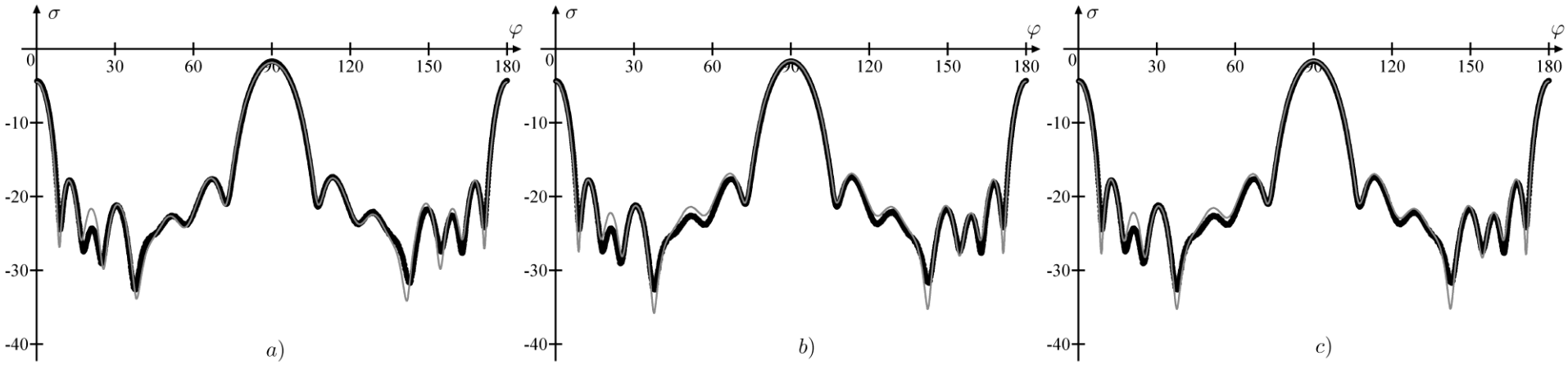
n_p	1	2	4	8	16	32	64	128
	1.00	1.91	3.39	6.22	11.07	19.95	30.52	42.10

Table 3. Acceleration of system solution for various number of processors. Number of cells 273600. Frequency 16 GHz.

n_p	1	2	4	8	16	32	64	128
	1.00	1.83	3.17	3.40	4.46	6.39	6.81	6.29

DIAGRAM OF BACKSCATTERING

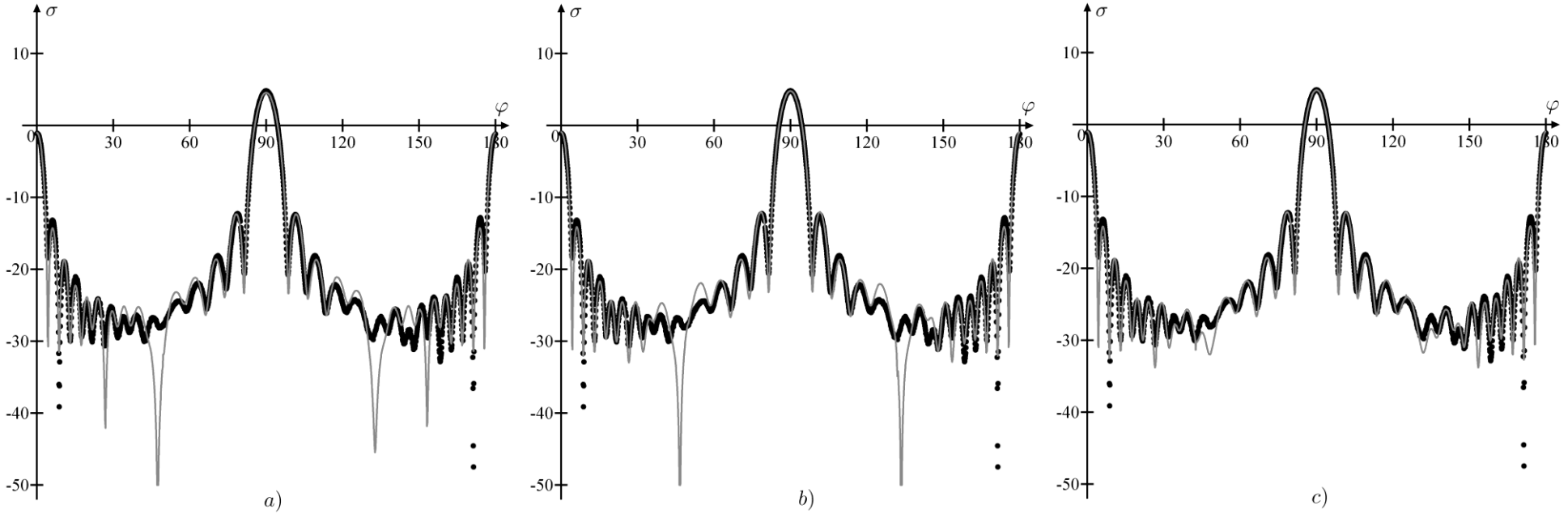
Radiation frequency 4 GHc - $\lambda = 7.5cm = 0.3H$



a) 13482 cells
$(h = 0.5cm)$

b) 21760 cells
$(h = 0.375cm)$

c) 45784 cells
$(h = 0.2cm)$



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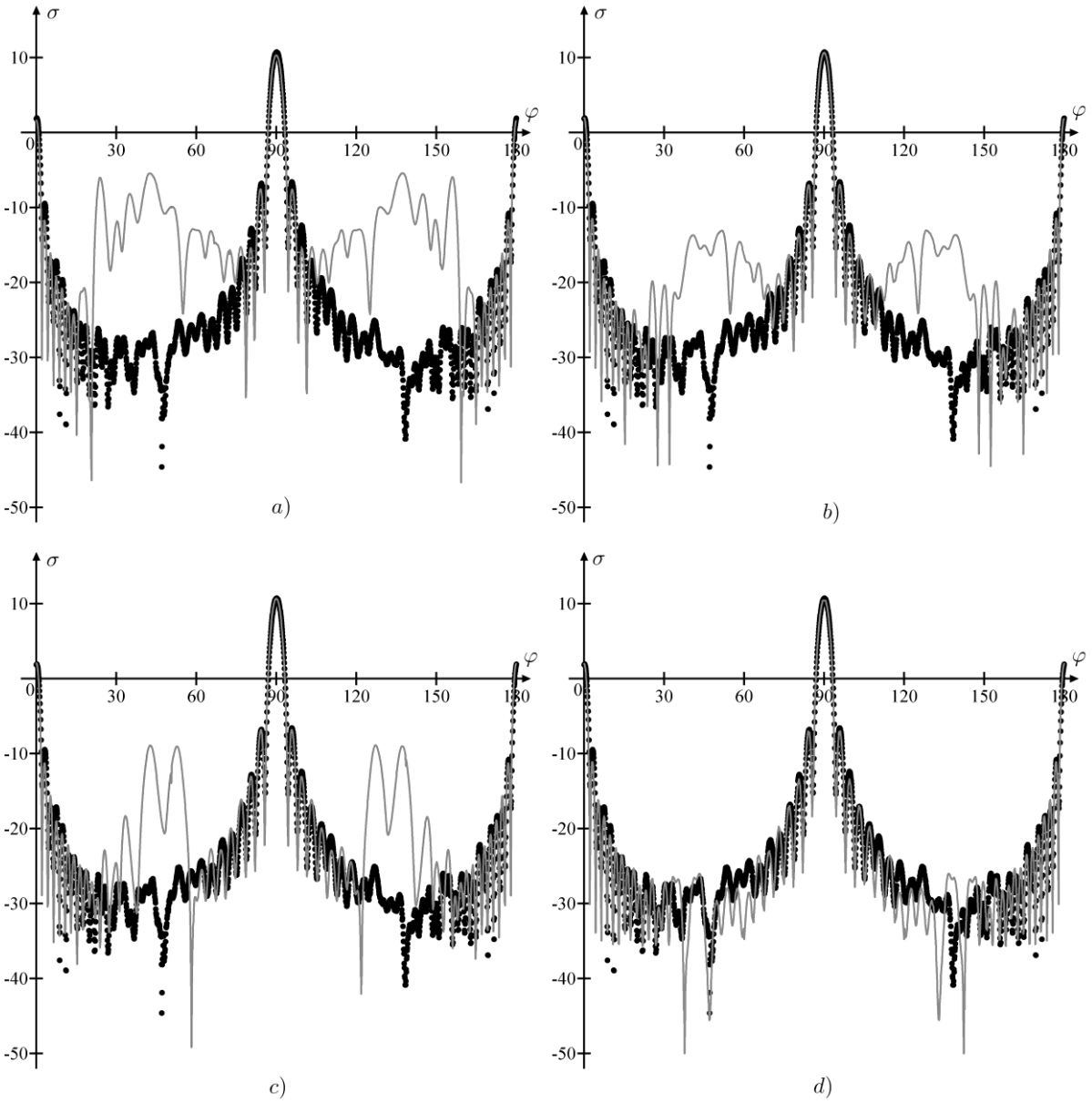

a) 13482 cells ($h = 0.5cm$)

b) 21760 cells ($h = 0.375cm$)

c) 45784 cells ($h = 0.2cm$)

d) 273600 cells ($h = 0.1cm$)

COMPUTING COSTS

13482 cells	Personal Computer, < 1 hour
21760 cells	Personal Computer, ~ 24 hour, RAM 36 GGb
45784 cells	а) with mosaic-skeleton method: Personal Computer, < 4 hour б) on supercomputer "Chebyshev" in Lomonosov Moscow State University supercomputer center, 150 processors and 225 Gb of RAM.
273600 cells	with mosaic-skeleton parallel method INM RAS computer cluster, 128 processors, ~ 8 hour